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Crossing of the cosmological constant boundary—an equation of state description

Hrvoje Štefanić¹

Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Av Diagonal 647, 08028 Barcelona, Catalonia, Spain

E-mail: stefanic@ecm.ub.es

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Abstract

The phenomenon of the dark energy transition between the quintessence regime ($w > -1$) and the phantom regime ($w < -1$), also known as the cosmological constant boundary crossing, is analysed in terms of the dark energy equation of state. It is found that the dark energy equation of state in the dark energy models which exhibit the transition is *implicitly* defined. The generalizations of the models explicitly constructed to exhibit the transition are studied to gain insight into the mechanism of the transition. It is found that the cancellation of the terms corresponding to the cosmological constant boundary makes the transition possible.

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(Some figures in this article are in colour only in the electronic version)

Among many important cosmological problems, the phenomenon of the present, late-time accelerated expansion of the universe has come to the forefront of the observational and theoretical efforts in recent years. Apart from the exciting series of cosmological observational results confirming the accelerated character of the expansion of the universe [1–3], we are witnessing many theoretical endeavours aimed at explaining the features of the present expansion of the universe, as well as the revival of some longstanding problems in cosmology and high energy physics, such as the cosmological constant problem [4].

From the theoretical viewpoint, there is still no decisive insight into the nature of the accelerating mechanism. However, many promising models have been proposed to explain the acceleration in the universe's expansion. Some of the interesting approaches include the braneworld models and the modifications of gravity at cosmological scales. The most studied accelerating mechanism is the existence of a cosmic component with a negative pressure, a so-called *dark energy* component. Dark energy is a very useful concept since all our ignorance on

¹ On leave of absence from the Theoretical Physics Division, Rudjer Bošković Institute, Zagreb, Croatia.

the acceleration phenomenon is encoded into a single cosmic component. It can also be very useful as an effective description of other approaches to the explanation of the acceleration of the universe. Many models of the dark energy have been constructed so far, assigning to dark energy different properties. A very general classification of these models is possible with respect to the parameter w of the dark energy equation of state (EOS), $p_d = w\rho_d$, where p_d and ρ_d refer to the dark energy pressure and energy density, respectively.² The benchmark value for the parameter of the dark energy EOS is $w = -1$ which is characteristic of the cosmological constant (CC). A problem associated to the CC value predicted by high energy physics, i.e. its discrepancy of many orders of magnitude with the value inferred from observations, is notoriously difficult. Such a situation has stimulated the development of dynamical dark energy models. Some prominent dynamical models of dark energy such as *quintessence* [5], *k-essence* [6] or *Chaplygin gas* [7] are characterized by $w > -1$. On the other side of the CC boundary are located models of *phantom energy* [8], with the property $w < -1$. These models are characterized by some tension between a certain favour from the observational side and certain disfavour from the theoretical side. Namely, the present attempts of the consistent formulation of phantom models face instabilities at the classical and the quantum level. Examples of such instabilities are the instability to the formation of gradients [9] or the quantum vacuum decay of phantom theories (resulting in very low limits on the cutoff of the effective phantom field models) [10]. The phantom-like dark energy is, however, observationally allowed and further study and development of (effective) phantom models free of the aforementioned instabilities is of special interest.

Many recent analyses of observational data [11], using ingenious parametrizations for the redshift dependence $w(z)$, show that the best fit values imply the transition of the dark energy parameter of EOS from $w > -1$ to $w < -1$ at a small redshift. This phenomenon has been referred to in the literature as *the crossing of the CC boundary*, *crossing of the phantom divide* or *the transition between the quintessence and phantom regimes*. It is important to stress that currently some other options, like that of the Λ CDM cosmology, are also consistent with the observational data. Should the future observations confirm the present indications of the crossing, the aspects of the theoretical description of the crossing might provide a useful means of distinguishing and discriminating various dark energy models and other frameworks designed to explain the present cosmic acceleration. Therefore, the crossing of the CC boundary is to some extent observationally favoured and its description is a theoretical challenge.

A number of approaches have been adopted so far to describe the phenomenon of the CC boundary crossing [12]. In our considerations of the phenomenon of the CC boundary crossing [13], we assume that the dark energy is a single, noninteracting cosmic component. We focus on the question whether the CC boundary crossing can be described using the dark energy EOS and if the answer is yes, which form the dark energy EOS needs to have to make the crossing possible. The equation of state is most frequently formulated as p given as an analytic expression of the energy density ρ . In the considerations given below we use a much broader definition of EOS. We define the equation of state *parametrically*, i.e. as a pair of quantities depending on the cosmic time ($\rho(t)$, $p(t)$), or equivalently on the scale factor a in the expanding universe ($\rho(a)$, $p(a)$). This definition easily comprises broad classes of dark energy models considered in the literature.

Let us start by considering a specific dark energy model which describes the CC boundary crossing. The dependence of the dark energy density on the scale factor in this model is given by

² Since the dark energy is the only component discussed in this paper, the subscripts d will be suppressed hereon.

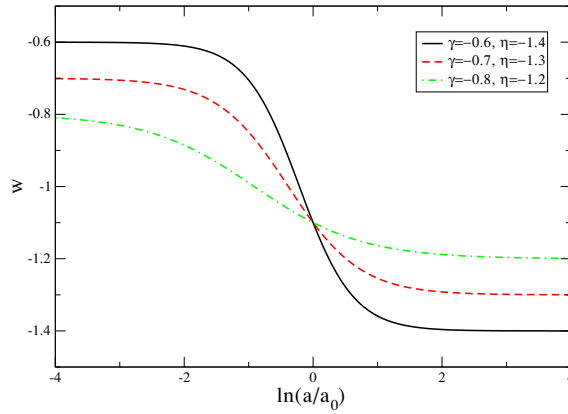


Figure 1. The dark energy parameter of EOS w given by (3) as a function of the scale factor a for the present value $w_0 = -1.1$ and three sets of parameters γ and η .

$$\rho = C_1 \left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + C_2 \left(\frac{a}{a_0}\right)^{-3(1+\eta)} . \tag{1}$$

where $\gamma > -1$ and $\eta < -1$. The scaling of this energy density resembles the sum of two independent cosmic components. However, we consider it to be the energy density of a *single* cosmic component and study its properties. Using the energy–momentum tensor conservation the expression for the dark energy pressure is obtained:

$$p = \gamma C_1 \left(\frac{a}{a_0}\right)^{-3(1+\gamma)} + \eta C_2 \left(\frac{a}{a_0}\right)^{-3(1+\eta)} . \tag{2}$$

Combining (1) and (2), the expression for the parameter of the dark energy EOS acquires the following form:

$$w = \frac{\gamma + \eta \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0}\right)^{3(\gamma - \eta)}}{1 + \frac{\gamma - w_0}{w_0 - \eta} \left(\frac{a}{a_0}\right)^{3(\gamma - \eta)}} . \tag{3}$$

The functional dependence of the parameter w on the scale factor is depicted in figure 1. Equations (1) and (2) can be further used to obtain the equation of state of the studied dark energy model:

$$\frac{p - \eta\rho}{(\gamma - \eta)C_1} = \left(\frac{\gamma\rho - p}{(\gamma - \eta)C_2}\right)^{(1+\gamma)/(1+\eta)} . \tag{4}$$

The most important feature of the obtained EOS is that it is defined *implicitly*. This result, obtained by the explicit construction, indicates that the phenomenon of the CC boundary crossing can be studied using the implicitly defined dark energy EOS. In the second reference of [14] it has been shown that the implicit dark energy EOS leads to the multivalued expressions for p in terms of ρ at the CC boundary crossing point.

Apart from the implicit character of the dark energy EOS that allows the CC boundary crossing, it would be of interest to gain additional insight into the mechanism of the crossing, i.e. the conditions necessary for the crossing to happen. To gain such an insight, we further consider the following dark energy EOS:

$$A\rho + Bp = (C\rho + Dp)^\alpha , \tag{5}$$

where A, B, C, D and α are the real parameters. The EOS (5) is a generalization of (4) and contains it as a special case. On the other hand, another choice of parameters leads to EOS of the form $p = -\rho - K\rho^\delta$ [14] which does not exhibit the CC boundary crossing. The generalized model (5) exhibits the crossing only for some parameter values and is therefore suitable for the study of the necessary conditions for the crossing. The dark energy density can be expressed in terms of the parameter w

$$\rho = \frac{(C + Dw)^{\alpha/(1-\alpha)}}{(A + Bw)^{1/(1-\alpha)}}, \quad (6)$$

which leads to the equation of evolution of w with the scale factor a :

$$\left(\frac{\alpha}{(F+w)(1+w)} - \frac{1}{(E+w)(1+w)} \right) dw = 3(\alpha - 1) \frac{da}{a}, \quad (7)$$

where abbreviations $E = A/B$ and $F = C/D$ have been introduced. A closer inspection of this equation reveals that there are several important values of w determining its evolution: $w = -1$, $w = -F$ and $w = -E$. Whenever these values *exist* in the description of the problem, they represent boundaries which cannot be crossed at a finite scale factor value and can only be approached asymptotically during the evolution of the universe. This simple observation already signals that, in order to have the transition of the CC boundary, the term corresponding to the $w = -1$ boundary has to be removed from (7).

The solution of (7) for the most interesting case when $E \neq -1$ and $F \neq -1$ has the form

$$\left| \frac{w+F}{w_0+F} \right|^{\alpha/(1-F)} \left| \frac{w+E}{w_0+E} \right|^{-1/(1-E)} \left| \frac{1+w}{1+w_0} \right|^{1/(1-E)-\alpha/(1-F)} = \left(\frac{a}{a_0} \right)^{3(\alpha-1)}. \quad (8)$$

This solution indicates that each of the potential boundaries can be removed by the suitable choice of the parameter α . The boundary at $w = -F$ is removed when $\alpha = 0$ and the boundary at $w = -E$ is removed when $\alpha \rightarrow \pm\infty$. The crossing of the CC boundary becomes possible with the choice $\alpha_{\text{cross}} = (1-F)/(1-E)$, i.e. for this value of the parameter α the CC boundary is removed. Equation (8) then becomes

$$\left| \frac{w+F}{w_0+F} \right| \left| \frac{w+E}{w_0+E} \right|^{-1} = \left(\frac{a}{a_0} \right)^{3(E-F)}, \quad (9)$$

which describes the transition from the $w > -1$ regime to the $w < -1$ regime.

An additional insight into the crossing mechanism can be obtained if equation (8) is studied in the rearranged form:

$$\frac{w + \frac{\alpha E - F}{\alpha - 1}}{(F+w)(E+w)(1+w)} dw = 3 \frac{da}{a}. \quad (10)$$

The numerator of the expression on the left-hand side can also be written as $w - w_*$, where $w_* = -(\alpha E - F)/(\alpha - 1)$. For specific values of the parameter α , the parameter w_* can become equal to $-F$ (for $\alpha = 0$), $-E$ (for $\alpha \rightarrow \pm\infty$) or -1 (for $\alpha_{\text{cross}} = (1-F)/(1-E)$). Therefore, for a specific value of the parameter α the terms corresponding to some of the boundaries get cancelled. This cancellation is a mathematical description of the mechanism behind the CC boundary transition.

A more general model of dark energy capable of transiting between the quintessence and phantom regimes can be constructed. We consider a model with the following scaling of the dark energy density:

$$\rho = \left(C_1 \left(\frac{a}{a_0} \right)^{-3(1+\gamma)/b} + C_2 \left(\frac{a}{a_0} \right)^{-3(1+\eta)/b} \right)^b. \quad (11)$$

This model can describe transitions of the CC boundary in both directions, i.e. from $w > -1$ to $w < -1$ and vice versa, depending on the sign of the parameter b , see figure 2. The dark energy pressure has the following form:

$$p\rho^{(1-b)/b} = \gamma C_1 \left(\frac{a}{a_0}\right)^{-3(1+\gamma)/b} + \eta C_2 \left(\frac{a}{a_0}\right)^{-3(1+\eta)/b}. \tag{12}$$

From (11) and (12), the dark energy EOS follows directly:

$$\frac{p - \eta\rho}{(\gamma - \eta)C_1} = \rho^{((1-b)(\gamma-\eta))/(b(1+\eta))} \left(\frac{\gamma\rho - p}{(\gamma - \eta)C_2}\right)^{(1+\gamma)/(1+\eta)}. \tag{13}$$

Starting from this explicitly constructed model, it is interesting to study its generalization in the form

$$A\rho + Bp = (C\rho + Dp)^\alpha (M\rho + Np)^\beta, \tag{14}$$

where A, B, C, D, M, N, α and β are the real coefficients and study the conditions for the CC boundary crossing within this generalization. Following the similar procedure as in the case of model (5), the evolution law for the dark energy parameter of EOS acquires the form

$$\left(\frac{\alpha D}{C + Dw} + \frac{\beta N}{M + Nw} - \frac{B}{A + Bw}\right) \frac{dw}{1 + w} = 3(\alpha + \beta - 1) \frac{da}{a}. \tag{15}$$

The solution of this equation in the most interesting case $A \neq B, C \neq D$ and $M \neq N$ is

$$\left|\frac{C + Dw}{C + Dw_0}\right|^{-\alpha D/(C-D)} \left|\frac{M + Nw}{M + Nw_0}\right|^{-\beta N/(M-N)} \left|\frac{1 + w}{1 + w_0}\right|^{\alpha D/(C-D) + \beta N/(M-N) - B/(A-B)} \times \left|\frac{A + Bw}{A + Bw_0}\right|^{B/(A-B)} = \left(\frac{a}{a_0}\right)^{3(\alpha + \beta - 1)}. \tag{16}$$

This solution for the scaling of w with a reveals that any of the boundaries $-A/B, -C/D, -M/N$ or -1 can be removed by the appropriate choice of parameters α and/or β . Therefore, the crossing of the CC boundary is possible in this generalized model if the exponent of $|1 + w|$ vanishes. This requirement can be expressed as a condition on one of the parameters α or β .

Using the insight gained from the studies of the models which are explicitly constructed to exhibit the transition and their generalizations, it is possible to study a model with a nontrivial implicitly defined equation of state of the form

$$A\rho^{2n+1} + Bp^{2n+1} = (C\rho^{2n+1} + Dp^{2n+1})^\alpha \tag{17}$$

and to show that the dark energy model characterized by this EOS is capable of describing the CC boundary crossing. To demonstrate the possibility of the aforementioned transition within the model (17), we show that a suitable choice of the parameter α can remove the $w = -1$ boundary from the problem. The evolution equation for the parameter of EOS is

$$\frac{w^{2n+1} + (\alpha E - F)/(\alpha - 1)}{(F + w^{2n+1})(E + w^{2n+1})} \frac{w^{2n}}{1 + w} dw = 3 \frac{da}{a}. \tag{18}$$

where $E = A/B$ and $F = C/D$. Choosing $(\alpha E - F)/(\alpha - 1) = 1$ the term corresponding to the CC boundary is removed since

$$\frac{w^{2n+1} + 1}{w + 1} = \xi(w) = \sum_{l=0}^{2n} (-w)^l \tag{19}$$

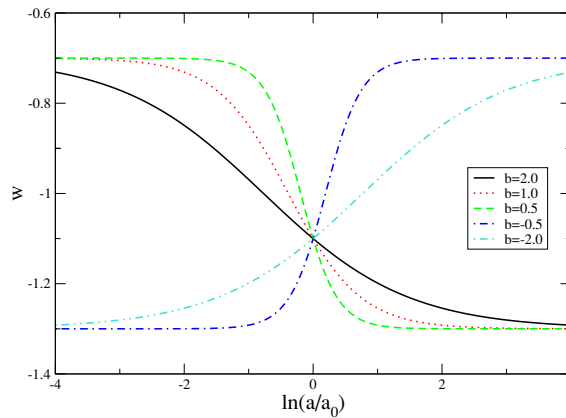


Figure 2. The parameter of the dark energy EOS w as a function of the scale factor of the universe a for the model (11). The positive values of the parameter b describe the transition from γ to η with the expansion of the universe, whereas the negative values for b describe the transition from η to γ as the universe expands. The values of the parameters used are $w_0 = -1.1$, $\gamma = -0.7$ and $\eta = -1.3$.

and $\xi(w)$ has no real roots. Equation (18) acquires the form

$$\frac{\xi(w)}{(F + w^{2n+1})(E + w^{2n+1})} w^{2n} dw = 3 \frac{da}{a}, \quad (20)$$

which describes smooth transitions between $w = -E^{-1/(2n+1)}$ and $w = -F^{-1/(2n+1)}$.

Finally, let us study the possibility of the CC boundary crossing in the dark energy model defined by the following EOS:

$$\rho = a e^{bp/\rho} (c - p/\rho)^\alpha. \quad (21)$$

The equation for the variation of w with a is

$$\left(b - \frac{\alpha}{1+c}\right) \frac{dw}{1+w} - \frac{\alpha}{1+c} \frac{dw}{c-w} = -3 \frac{da}{a}. \quad (22)$$

The choice $b = \alpha/(1+c)$ removes the CC boundary and results in the following solution for the dark energy parameter of EOS:

$$w = c - (c - w_0) \left(\frac{a}{a_0}\right)^{-3(1+c)/\alpha}, \quad (23)$$

which describes the crossing of the CC boundary. The choice of the parameters $\alpha < 0$, $w_0 < -1$ and $-1 < c < -1/3$ yields a transition from the quintessence regime to the phantom regime at a positive redshift.

In the generalized models of this paper used to study the CC boundary crossing, special conditions need to be met for the crossing to occur. Namely, one of the model parameters needs to acquire a value determined by the other parameter values. In a sense, if a parametric space of a model is D dimensional, the set of parameter values for which the transition occurs is $(D - 1)$ dimensional. Therefore, it has been shown that the CC boundary crossing can be described in terms of EOS, but that the model parameters need to be chosen in a special way. However, a more extensive analysis of the dark energy models with an implicitly defined EOS is needed to verify if this is a general feature of these models.

In conclusion, the dark energy transition between the quintessence and the phantom regimes is studied using the dark energy EOS. It is found that in models which exhibit

the CC boundary crossing, the EOS is implicitly defined. Many models described in this paper mainly serve as an illustration of the possibility of description of the CC boundary crossing using the implicitly defined dark energy EOS. The selection of the realistic implicit dark energy EOS would require the comparison with the observational data [11]. Within the generalized models the crossing is possible when there is the cancellation of the terms corresponding to the CC boundary. The CC boundary crossing requires a special choice of model parameters and therefore the study of its aspects might be useful in discriminating the crossing in noninteracting dark energy models from the cosmological models where the CC boundary crossing is an effective phenomenon, see e.g. [15].

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